Cooperative Game Theoretic Models to Analyze Multi-class Queueing Systems

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Game Theory and Applications

(GTA 2022)

Agenda

- ***Introduction***
- ***Objective***
- ***Game Theoretic Representation***
- ***Methodology***
- ***Results***
- ***Conclusion and future research***
- ***References***

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- *Some of the works are* ***Liu and Yu (2022) , Armony et al. (2021) , Yu et al. (2015), Anily and Haviv (2010).***

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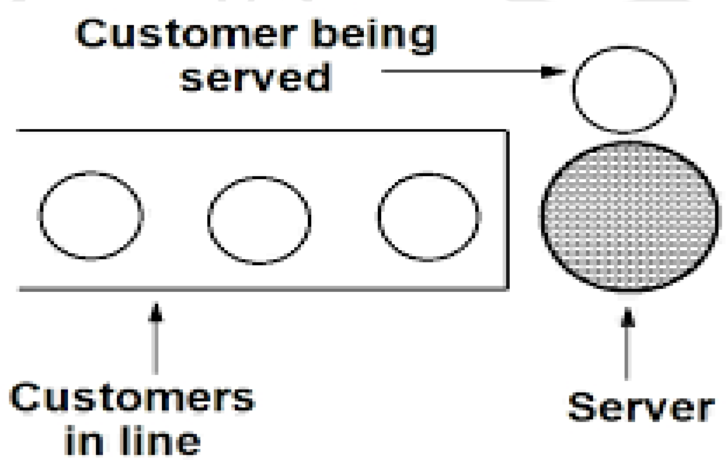

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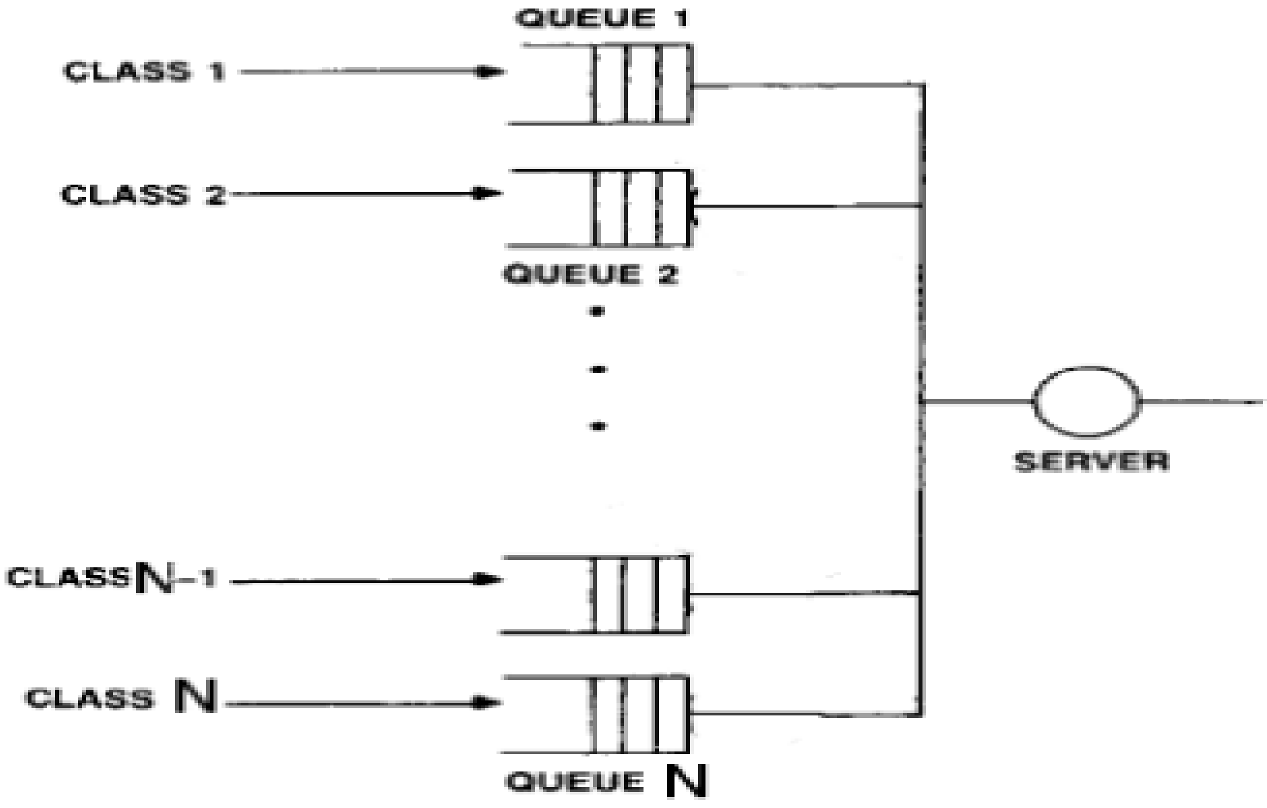

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- *Now we will establish worth functions v(.) of any player (i), any coalition (S) and the grand coalition (N)*
- *Worth function of the grand coalition, v({N}) must follow Kleinrock's Conservation law (Kleinrock, 1965) where, right hand side is independent of any scheduling policy π,*
- $\sum_{i=1}^{N} \rho_i W_i^{\pi} = \frac{\rho W_0}{(1-\rho)}$ *. Here,* ρ_i *is load factor of class i,* W_i *is mean waiting time of class i and* $W_0 = \sum_{i=1}^{N} \frac{\lambda_i}{2} [\sigma_i^2 + \frac{1}{\mu_i^2}]$

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- *V({N}) is independent of scheduling policies.*

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- *The Core,* $C(P, v) = \{(x_1, x_2)\text{:}\ \frac{\rho_1 W_0}{(1-\rho_1)} \leq x_1 \leq \frac{\rho_1 W_0}{(1-\rho_2)(1-\rho)}\ ;\ x_1 + x_2 = \frac{\rho W_0}{(1-\rho)}\}$

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- *Shapley Value belongs to the Core*

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- $\hat{\varphi}_1 = \frac{W_0 [\rho\rho_2 + 2(1-\rho)]}{2(1-\rho_1)(1-\rho_2)(1-\rho)}$ *and* $\hat{\varphi}_2 = \frac{W_0 [\rho\rho_1 + 2(1-\rho)]}{2(1-\rho_1)(1-\rho_2)(1-\rho)}$

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- *Server selects the highest priority class*

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- *We found $\beta^{Shapley}$ which can allocate $\hat{\varphi}_1$ and $\hat{\varphi}_2$ fairly*
- $\beta^{Shapley} = \frac{(2-\rho)(1-\rho_1)}{\rho\rho_1+2(1-\rho)}\ 1_{\{\rho_1\geq\rho_2\}} + \frac{\rho\rho_2+2(1-\rho)}{(2-\rho)(1-\rho_2)}\ 1_{\{\rho_1<\rho_2\}}$

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- *At the extreme case when* $\rho_1 \rightarrow 1$ *then* $\beta^{Shapley} \rightarrow 0$. *This implies static high priority to class 1*

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Load factor of both classes are equal

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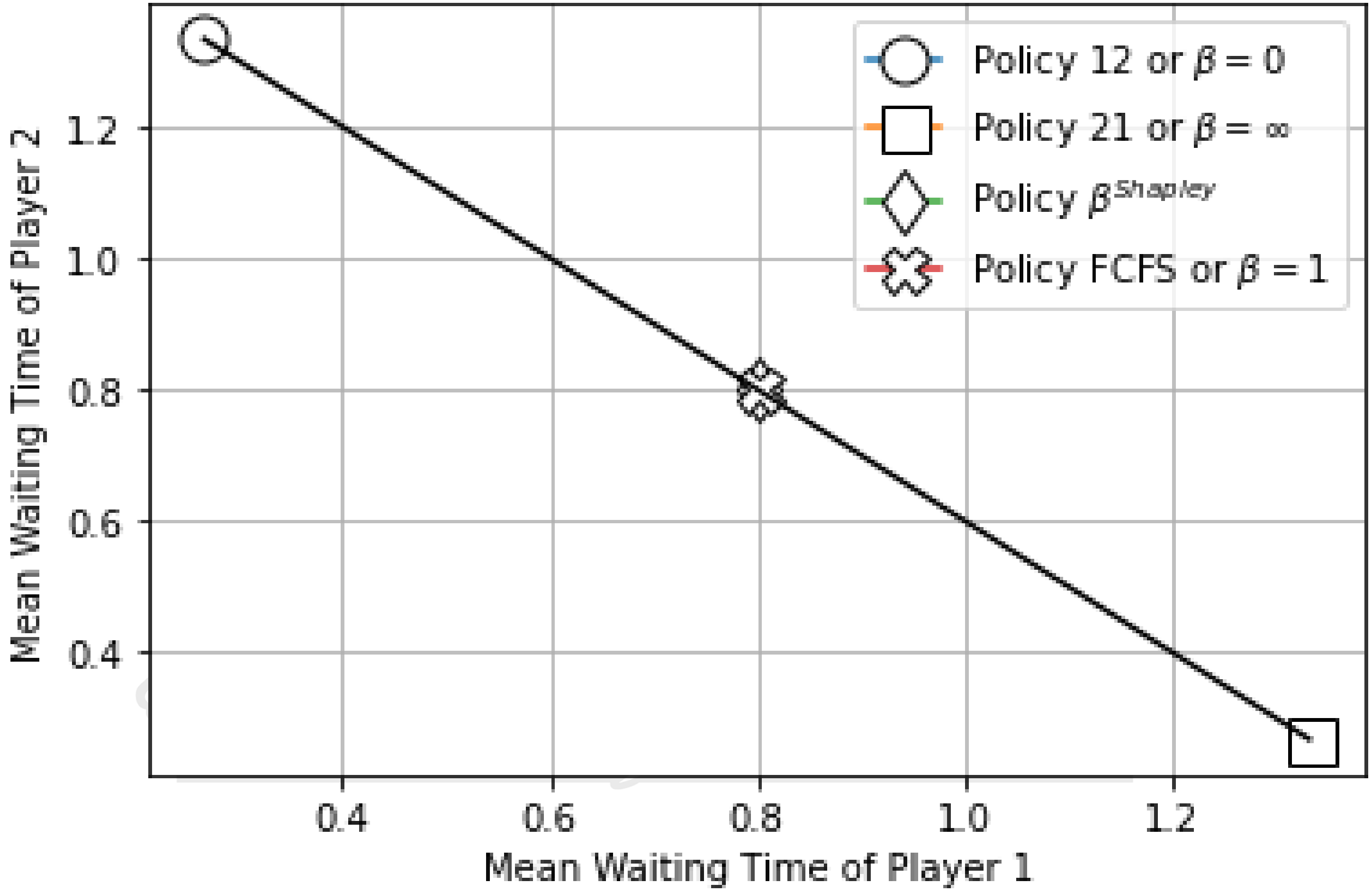
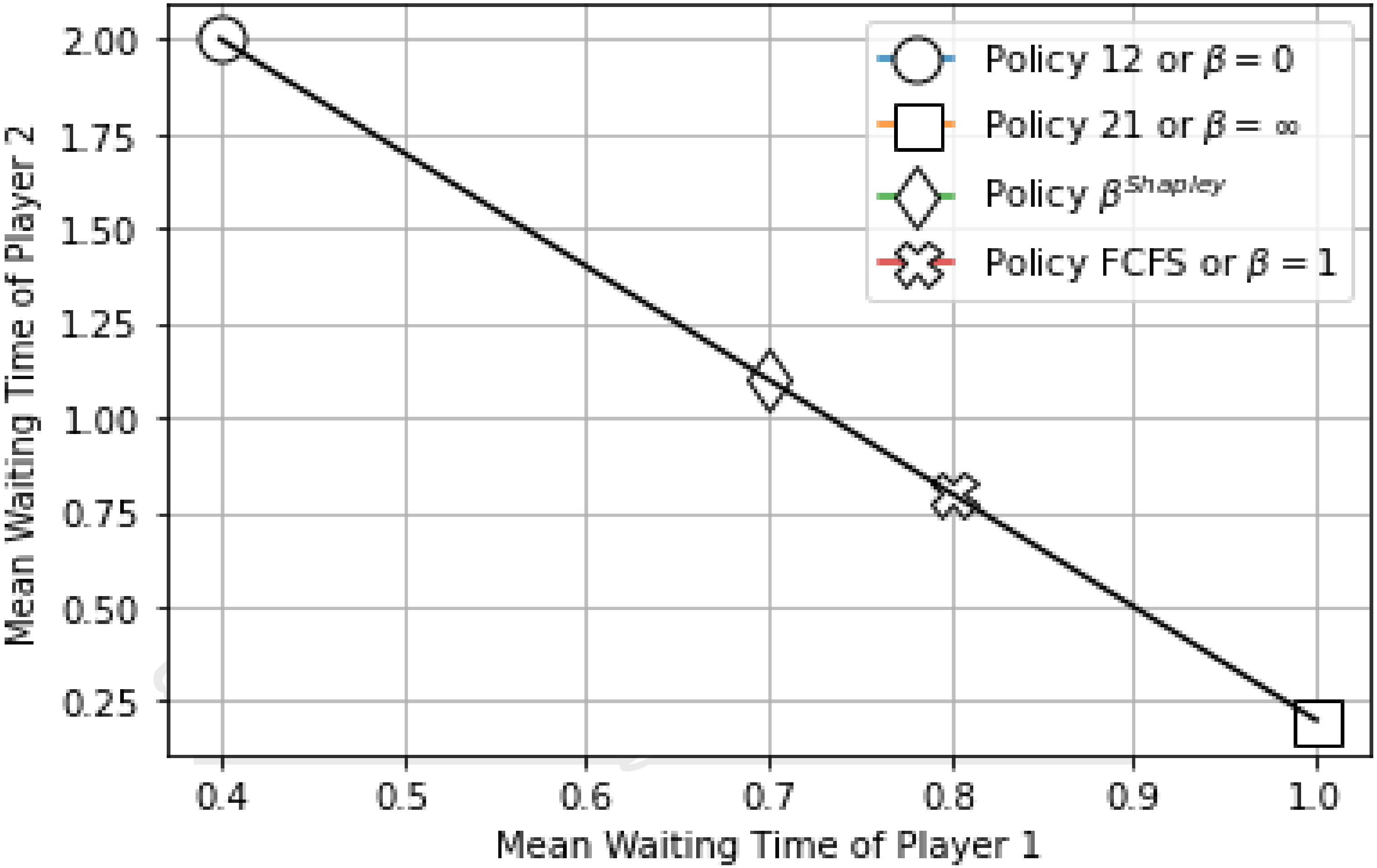


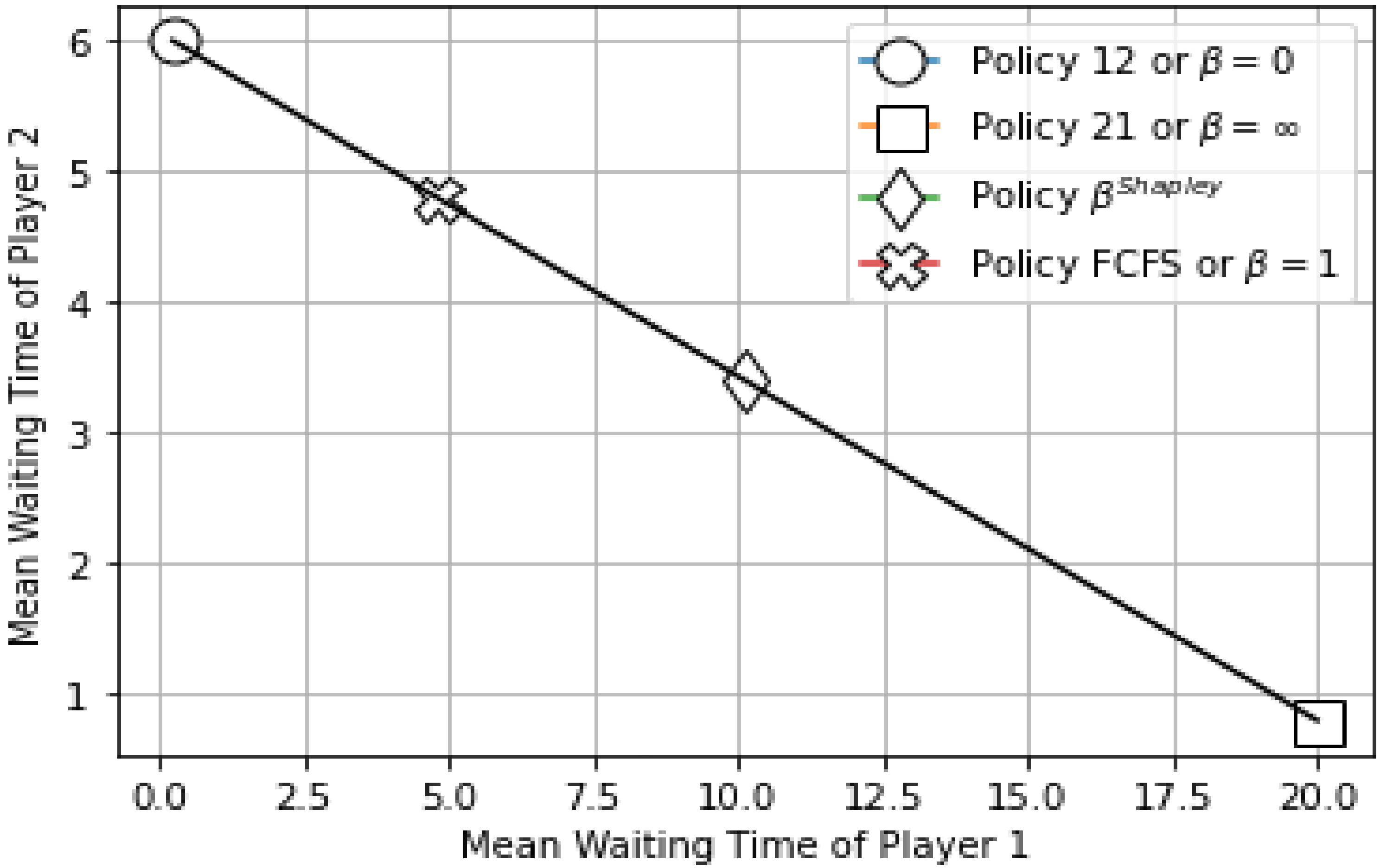
Load factor of class 1 is higher

Load factor of class 1 is higher



Load factor of class 2 is higher

Load factor of class 2 is higher



Conclusion and Future Research

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- *To explore cooperative game theoretic solutions such as Shapley Values, The Core, Nucleolus for N-class game.*

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- *To explore fair scheduling policy for N-class game.*
- *To design the game by considering other values of β (except 0 and ∞)*

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Thank You.